Find the rectangular equation for the surface by eliminating the parameter from the vector-valued function.

1) $\vec{\mathbf{r}}(u,v) = u\,\mathbf{i} + v\,\mathbf{j} + \frac{v}{2}\mathbf{k}$

$$\boxed{y - 2z = 0}$$

2) $\vec{\mathbf{r}}(u,v) = 2u\cos v\,\mathbf{i} + 2u\sin v\,\mathbf{j} + \frac{1}{2}u^2\,\mathbf{k}$

$$\boxed{z = \frac{1}{8}(x^2 + y^2)}$$

3) $\vec{\mathbf{r}}(u,v) = 2u\cos v\,\mathbf{i} + 2u\sin v\,\mathbf{j} + v\,\mathbf{k}$

$$\boxed{\tan z = \frac{y}{x}}$$

Find a parametric representation for the surface.

4) The plane $z = y$.

$$\boxed{\vec{\mathbf{r}}(u,v) = u\,\mathbf{i} + v\,\mathbf{j} + v\,\mathbf{k}}$$

5) The plane that passes through the point $(1,\ 2,\ -3)$ and contains the vectors $\mathbf{i}+\mathbf{j}-\mathbf{k}$ and $\mathbf{i}-\mathbf{j}+\mathbf{k}$.

$$\boxed{\vec{\mathbf{r}}(u,v)=(1+u+v)\,\mathbf{i}+(2+u-v)\,\mathbf{j}+(-3-u+v)\,\mathbf{k}}$$

6) The cone $x=\sqrt{16y^2+z^2}$

$$\boxed{\vec{\mathbf{r}}(y,z)=\sqrt{16y^2+z^2}\,\mathbf{i}+y\,\mathbf{j}+z\,\mathbf{k}\quad\text{or}\quad\vec{\mathbf{r}}(u,v)=u\,\mathbf{i}+\frac{1}{4}u\cos v\,\mathbf{j}+u\sin v\,\mathbf{k}\quad u\ge 0,\ \ 0\le v\le 2\pi}$$

7) The part of the sphere $x^2+y^2+z^2=4$ that lies above the cone $z=\sqrt{x^2+y^2}$

$$\vec{\mathbf{r}}(x,y)=x\,\mathbf{i}+y\,\mathbf{j}+\sqrt{4-x^2-y^2}\,\mathbf{k}\ \text{ where } x^2+y^2\le 2$$

or

$$\vec{\mathbf{r}}(u,v)=2\sin u\cos v\,\mathbf{i}+2\sin u\sin v\,\mathbf{j}+2\cos u\,\mathbf{k}\quad 0\le u\le\frac{\pi}{4},\ \ 0\le v\le 2\pi$$

8) The part of the paraboloid $z = x^2 + y^2$ that lies inside the cylinder $x^2 + y^2 = 9$

$$\boxed{\vec{\mathbf{r}}(u,v) = v\cos u\,\mathbf{i} + v\sin u\,\mathbf{j} + v^2\,\mathbf{k} \quad 0 \le v \le 3}$$

Write a set of parametric equations for the surface of revolution obtained by revolving the graph of the function about the given axis.

9) $y = \sqrt{x}, \quad 0 \le x \le 6 \quad x$-axis

$$\boxed{\vec{\mathbf{r}}(u,v) = u\,\mathbf{i} + \sqrt{u}\cos v\,\mathbf{j} + \sqrt{u}\sin v\,\mathbf{k} \quad 0 \le u \le 4 \quad 0 \le v \le 2\pi}$$

10) $x = \sin z, \quad 0 \le z \le \pi \quad z$-axis

$$\boxed{\vec{\mathbf{r}}(u,v) = \sin u\cos v\,\mathbf{i} + \sin u\sin v\,\mathbf{j} + u\,\mathbf{k} \quad 0 \le u \le \pi \quad 0 \le v \le 2\pi}$$

11) $z = y^2 + 1, \quad 0 \le y \le 2 \quad y$-axis

$$\boxed{\vec{\mathbf{r}}(u,v) = \left(u^2+1\right)\cos v\,\mathbf{i} + u\,\mathbf{j} + \left(u^2+1\right)\sin v\,\mathbf{k} \quad 0 \le u \le 2 \quad 0 \le v \le 2\pi}$$

Find an equation of the tangent plane to the given parametric surface at the specified point.

12) $\vec{\mathbf{r}}(u,v) = u^2\,\mathbf{i} + v^2\,\mathbf{j} + uv\,\mathbf{k} \quad u = 1, \quad v = 1$

$$\boxed{x + y - 2z = 0}$$

13) $\vec{\mathbf{r}}(u,v) = uv\,\mathbf{i} + u\sin v\,\mathbf{j} + v\cos u\,\mathbf{k} \quad u = 0, \quad v = \pi$

$$\boxed{y = 0}$$

Find the area of the surface

14) The part of the plane $x + 2y + z = 4$ that lies inside the cylinder $x^2 + y^2 = 4$.

$$\boxed{4\pi\sqrt{6}}$$

15) The part of the surface $z = 1 + 3x + 2y^2$ that lies above the triangle with vertices $(0, 0)$. $(0, 1)$, and $(2, 1)$.

$$\boxed{\frac{1}{24}\left(26^{3/2} - 10^{3/2}\right)}$$

16) The part of the hyperbolic paraboloid $z = y^2 - x^2$ that lies between the cylinders $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$

$$\boxed{\frac{\pi}{6}\left(17\sqrt{17} - 5\sqrt{5}\right)}$$

17) Find the surface area of the torus given by $\vec{\mathbf{r}}(u, v) = (2 + \cos u)\cos v\,\mathbf{i} + (2 + \cos u)\sin v\,\mathbf{j} + \sin u\,\mathbf{k}$.

$$\boxed{8\pi^2}$$